Fundamentals of Algorithms
CS502-Spring 2011

SOLUTION ASSIGNMENT #2

Deadline

Your assignment must be uploaded/submitted at or before **6th May 2011**

Uploading instructions

Please view the **assignment submission process** document provided to you by the Virtual University to upload the assignment.

Rules for Marking

It should be clear that your assignment will not get any credit if:

- o The assignment is submitted after due date.
- o The submitted assignment does not open or run.
- o **The assignment is copied.**

Objectives

This assignment will help you to understand the concept of recurrence relations and way to solve then and writing asymptotic notation after analyzing and solving recurrences .The other main focus is to learn dynamic programming applications and edit distance problem solution using dynamic programming technique which will be ultimately enhance your vision and logics to think critically and analytically.

Guidelines

1. In order to attempt this assignment you should have full command on Lecture #8 ,9 and Lecture # 17,18

2. In order to solve this assignment you have strong concepts about following topics

- ✓ Recurrence Relations and their solutions
- ✓ Dynamic programming technique and Edit distance problem

3. Normally these formulas are very handy:

If $x^y = z$ then $y = \log_x z$

$$a^{\log_b n} = n^{\log_b a}$$

Also

$$\sum_{i=1}^{n} a_i = \frac{n}{2}(a_1 + a_n) \qquad \sum_{i=1}^{n} i = \frac{n}{2}(n+1) \qquad \sum_{k=0}^{m} r^k = \frac{1-r^{m+1}}{1-r}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \text{ (for n >= 1)}$$

Books to read for solution

Cormen, Leiserson, Rivest, and Stein (CLRS) 2001, **Introduction to Algorithms**, (2nd ed.) McGraw Hill.

Estimated Time 4 hours

Your concepts and logics will take actual measure of time ;however first question should not take more than 1.5 hour and for question 2 you may solve in 2.5 hours It all depends upon your sheer concentration.

Question# 1 **(10)**

After analyzing the pseudo code for an algorithm following recurrence relation have been developed you are required to solve this recurrence relation using iterative method and make proper assumptions for final solution and give answer at end in asymptotic form.

$$T(n) = \begin{Bmatrix} 1 \quad if \quad n = 1 \\ 5T(n/6) \quad + \quad n \quad if \quad n > 1 \end{Bmatrix}$$

Assume n to be a power of 6, i.e., $n = 6^k$ and $k = \log_6 n$

This assumption is to make the logic smooth for the series calculation as other all possibilities will be covered in it

.

$$T(n)=5T\left(\frac{n}{6}\right)+n \ \text{ Given recurrence}$$

$$T(n)=5\left[5T\left(\frac{n}{36}\right)+n/6\right]+n$$

$$T(n)=25T\left(\frac{n}{36}\right)+5\left(\frac{n}{6}\right)+n$$

$$T(n)=25\left[5T\left(\frac{n}{216}\right)+\left(\frac{n}{36}\right)\right]+5\left(\frac{n}{6}\right)+n$$

$$T(n)=125T\left(\frac{n}{216}\right)+25\left(\frac{n}{36}\right)+5\left(\frac{n}{6}\right)+n$$

$$=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$T(n)=5^k+\left(\frac{n}{6^k}\right)+5^{k-1}+\left(\frac{n}{6^{k-1}}\right)+\ldots\ldots\ldots+125\left(\frac{n}{216}\right)+25\left(\frac{n}{36}\right)+5\left(\frac{n}{6}\right)+n$$

$$T(n)=5^k+\left(\frac{n}{6^k}\right)+\sum_{i=0}^{k-1}\frac{5^i}{6^i}n$$

$$n=6^k \ , \ \ T(1)=1$$

$$T(n)=5^k+\left(\frac{n}{6^k}\right)+\sum_{i=0}^{k-1}\frac{5^i}{6^i}n$$

$$T(n)=5^{\log_6 n}T(1)+\sum_{i=0}^{\log_6 n-1}\frac{5^i}{6^i}n$$

$$T(n)=n^{\log_6 5}+n\sum_{i=0}^{\log_6 n-1}\left(\frac{5}{6}\right)^i$$

$$\sum_{i=0}^{\log_6 n-1}\left(\frac{5}{6}\right)^i=\frac{\frac{5}{6}^{\log_6 n-1+1}-1}{\frac{5}{6}-1}$$

$$=\frac{\frac{5}{6}^{\log_6 n}-1}{-1/6}$$

$$T(n)=n^{\log_6 4}+n\left[\frac{\frac{5}{6}^{\log_6 n}-1}{-1/6}\right]$$

$$[\frac{5}{6}]^{\log_6 n}=n^{\left(\log_6\frac{5}{6}\right)}=n^{\log_6 5-\log_6 6}=n^{\log_6 5-1}=\frac{n^{\log_6 5}}{n}$$

finally

$$T(n)=n^{\log_6 5}+n\left(\frac{\frac{n^{\log_6 5}}{n}-1}{-1/6}\right)$$

$$T(n)=n^{\log_6 5}+\frac{n^{\log_6 5}-n}{-1/6}$$

$$T(n)=n^{\log_6 5}+6\left(n-n^{\log_6 5}\right)$$

$$T(n)=6n-5n^{\log_6 5}$$

$T(n)=6n-5n^{0.90}\in\Theta(n)$ So final complexity is linear time means $\Theta(n)$

Question# 2 **(10)**

Use the following dynamic programming based recurrence edit distance to find the possible edit scripts while converting “INVENTION “to “CONVENTION”.

$$E(i,j) = \min \begin{pmatrix} E(i-1,j)+1 & \\ E(i,j-1)+1 & \\ E(i-1,j-1)+1 & \text{if } A[i] \neq B[j] \\ E(i-1,j-1) & \text{if } A[i] = B[j] \end{pmatrix}$$

Solution Q#2

Some Basic assumptions:

E(i-1, j) + 1 repsents deletion and in table below "↓" has been used .

E(i, j-1) + 1 repsents insertion and in table "→"*has been* used .

E(i-1, j-1) + 1 if A[i]≠B[j] is for substitution and in table"↘" has been used.

E(i-1, j-1) if A[i]=B[j] is to maintain; means previous adjacent diagnol value is transfered with out any cost and here in table"↘"
has been used . Note here this symbol is being used for two purposes one for substitution adding "1" in last cost and other for "Maintain " without adding one .

Solution table

		C	O	N	V	E	N	T	I	O	N
	0	→1	→2	→3	→4	→5	→6	→7	→8	→9	→10
I	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4	↘ →5	↘ →6	↘ →7	↘ 7	→8	→9
N	↓ 2	↘↓ 2	↘ 2	↘ 2	→3	→4	↘ →5	→6	→7	↘ →8	↘ 8
V	↓ 3	↘↓ 3	↘↓ 3	↘↓ 3	↘ 2	→3	→4	→5	→6	→7	→8
E	↓ 4	↘↓ 4	↘↓ 4	↘↓ 4	↓ 3	↘ 2	→3	→4	→5	→6	→7
N	↓ 5	↘↓ 5	↘↓ 5	↘↓ 5	↘↓ 5	↓ 3	↘ 2	→3	→4	→5	↘ →6
T	↓ 6	↘ 6	↘↓ 6	↘↓ 6	↘↓ 6	↓ 4	↓ 3	↘ 2	→3	→4	→5
I	↓ 7	↘↓ 7	↘↓ 7	↘↓ 7	↘↓ 7	↓ 5	↓ 4	↓ 3	↘ 2	→3	→4
O	↓ 8	↘↓ 8	↘↓ 8	↘↓ 8	↘↓ 8	↓ 6	↓ 5	↓ 4	↓ 3	↘ 2	→3
N	↓ 9	↘↓ 9	↘↓ 9	↘↓ 9	↘↓ 9	↓ 7	↓ 6	↓ 5	↓ 4	↓ 3	↘ 2

Possible Edit Transcripts

Edit Transcript Path 1: Note here cell E(0,1) which give us insertion value at initial place and E(1,2) diagonal arrow gives us the substitution at second path all other values are maintained Path 1

1+	**1+**	**1+**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**=2**
I	**S**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	
_	**I**	**N**	**V**	**E**	**N**	**T**	**I**	**O**	**N**	
C	**O**	**N**	**V**	**E**	**N**	**T**	**I**	**O**	**N**	

Edit Transcript Path 2: Note here E (1, 1) diagonal arrow gives us substitution at first place and E (1, 2) horizontal arrow gives us insertion value at second place and all other values are maintained path 2

1+	**1+**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0=2**
S	**I**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	**M**
I	**_**	**N**	**V**	**E**	**N**	**T**	**I**	**O**	**N**
C	**O**	**N**	**V**	**E**	**N**	**T**	**I**	**O**	**N**

Conclusion: The best possible way can be done at the cost of "2" for the conversion of INVENTION to CONVENTION by the dynamic approach result.

BEST OF LUCK
PEACEFUL MINDS GIVE INNOVATIONS