MTH401 - Final term PAPER

SOLVED BY : AQUALEO | REMEMBER ME IN YOUR PRAYERS

Total Question: 52
Mcqz: 40
Subjective question: 12
4 q of 5 marks
4 q of 3 marks
4 q of 2 marks

Guidelines:
You will have to clear the concepts and formulas of topics according to which questions are solved in file.

TODAY's PAPER no 1

Objective: MCQz

Topic	**Number of Mcqz**
Ratio Test Convergence Divergence	5
D.E(Integrating Factors +Homogenous+linear+bernoli)	**7**
Z= $\sqrt{X^2 + Z^2}$	**1**
Reactance & Impedence	**1**
Damped Motion	**2**
Maxima	**1**
Quasi period	**3**
Besslen's Equation	**1**
Matrix Type(square+system to matrix conversion)	**6**
Eigen Values+Eigen Vector	**4**
Multiplicy of Eigen Vector	**3**

D.E operator	**2**
General Solution	**1**
BVP	**1**

Please review the formulas of above topics.

Q:1

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = 5e^{t}$$
$$\frac{dx}{dt} - x + \frac{dy}{dt} = e^{t}$$

lec 36 example 1

in decoupled form.

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = 5e^{t}$$
$$\frac{dx}{dt} - x + \frac{dy}{dt} = e^{t}$$

$$(2D-5)x + Dy = 5e^{t}$$
$$(D-1)x + Dy = e^{t}$$

Determinants are $\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix}$, $\begin{vmatrix} 5e^{t} & D \\ e^{t} & D \end{vmatrix}$, $\begin{vmatrix} 2D-5 & 5e^{t} \\ D-1 & e^{t} \end{vmatrix}$

Therefore, in decpoupled form, we get

$$\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix} x \quad \begin{vmatrix} 5e^{t} & D \\ e^{t} & D \end{vmatrix}$$

$$\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix} y \quad \begin{vmatrix} 2D-5 & 5e^{t} \\ D-1 & e^{t} \end{vmatrix}$$

Q:2

Find order of homogenous equation obtained from non homogenous differential equation:

$y'' + 4y' + 3y = 4x^2 + 5?$ **? (2 MARKS)**

Find the eigenvalues of the following system

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

Solution:

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

$$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$$(3-\lambda)(-3-\lambda) + 36 = 0$$

$$3(-3-\lambda) - \lambda(-3-\lambda) + 36 = 0$$

$$-9 - 3\lambda + 3\lambda + \lambda^2 + 36 = 0$$

$$-9 + \lambda^2 + 36 = 0$$

$$\lambda^2 + 27 = 0$$

$\lambda = \sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

Q:3

What is Chemical reaction first order equation? (2) Page no 100

Answer:

$$\frac{dX}{dt} = k\,X$$

$k < 0$ because X is decreasing.

Q:4

What is charachteristic equation? Page no 379

Answer:

$$\det(A - \lambda I) = 0$$

This equation is called the characteristic equation of the matrix A.

Q:5

Can we extend power series?

AnsweR: Page no 268

I answered in yes and then wrote the extended form of power series.

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots.$$

Q:6 Page no 371

Find the derivative and the integral of the following matrix

$$X(t) = \begin{pmatrix} \sin 2t \\ e^{3t} \\ 8t-1 \end{pmatrix}$$

Solution:

The derivative and integral of the given matrix are, respectively, given by

$$X'(t) = \begin{pmatrix} \frac{d}{dt}(\sin 2t) \\ \frac{d}{dt}(e^{3t}) \\ \frac{d}{dt}(8t-1) \end{pmatrix} = \begin{pmatrix} 2\cos 2t \\ 3e^{3t} \\ 8 \end{pmatrix}$$

Q:7

Write system of equation in matrix form?

Solution: Page no 387

$$\frac{dx}{dt} = -3x + 4y - 9z$$
$$\frac{dy}{dt} = 6x - y$$
$$\frac{dz}{dt} = 10x + 4y + 3z$$

Solution :

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Q:8 Page no 98

Dudce special case of logistic equation (epidemic spread)? (5)

The natural assumption is that the rate $\frac{dx}{dt}$ of spread of disease is proportional to the number $x(t)$ of the infected people and number $y(t)$ of people not infected people. Then

$$\frac{dx}{dt} = kxy$$

Since $x + y = n + 1$

Therefore, we have the following initial value problem

$$\frac{dx}{dt} = kx(n+1-x), \qquad x(0) = 1$$

The last equation is a **special case of the logistic equation** and has also been used for the **spread of information** and the **impact of advertising** in centers of population.

Q:9

Find order of homogenous equation obtained from non homogenous differential equation:

$y'' + 4y' + 3y = 4x^2 + 5$?? **(2 MARKS)**

Q:10:

Find a series solution for the differential equation $y'' + y = 0$ about $x_0 = 0$ such that

Find condition of cofficent for a_{n+2} & a_n (c_{n+2} & c_n) ?

Q:11

Which series is identically zero? Page no 273

Answer:

Series that are Identically Zero

If for all real numbers x in the interval of convergence, a power series is identically zero i.e.

$$\sum_{n=o}^{\infty} c_n (x-a)^n = 0, \quad R > 0$$

Then all the coefficients in the power series are zero. Thus we can write

$$c_n = 0, \quad \forall \ n = 0, 1, 2, \ldots$$

Q:12

$$A \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}$$

eigen values ?

Eigen vectors ?

Note: I am not going to solve this question solve it by your self by consulting two examples below.

========================First Paper End======================

Q1: Find Coefficient of metrix:

$$\frac{dx}{dt} = -3x - 2y$$

$$\frac{dy}{dt} = 5x + 7y$$

Solution:

Cofficent of matrix =

$$A \begin{bmatrix} -3 & -2 \\ 5 & 7 \end{bmatrix}$$

Q2: Eigen Values of metrics.

A= $\begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$

Consider the question below:

$$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$$(3-\lambda)(-3-\lambda) + 36 = 0$$

$$3(-3-\lambda) - \lambda(-3-\lambda) + 36 = 0$$

$$-9 - 3\lambda + 3\lambda + \lambda^2 + 36 = 0$$

$$-9 + \lambda^2 + 36 = 0$$

$$\lambda^2 + 27 = 0$$

$\lambda = \sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

This question is similar to above.

Q3: whether or not a singular points have real number if not then give some examples?

Answer: Page no 284

(b) The singular points need not be real numbers.
The equation $(x^2+1)y''+2xy'+6y=0$ has the singular points at the solutions of $x^2+1=0$, namely, $x=\pm i$.

Q4: Solve the differential equation. $\frac{1}{y}\frac{dy}{dx} \quad 1$

Solution:

$$\frac{1}{y}\frac{dy}{dx} \quad 1$$

$$\frac{dy}{y} \quad (1)dx$$

$$\int\frac{dy}{y} \quad \int(1)dx$$

$$\ln y = x + c$$

$$y \quad e^{x+c}$$

Q5: complementary solution of DE

$$y'' - 4y' + 4y = 2e^{2x}$$

Solution: Page no 182

Step 2 To find the complementary function, we consider

$$y'' - 2y' + y = 0$$

The auxiliary equation is

$$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1, 1$$

The complementary function for the given equation is

$$y_c = c_1 e^x + c_2 x e^x$$

Q6: state the Bessel's function of first kind of order ½ and -1/2.

Solution: Page no 313

$$J_v(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)\Gamma(1+v+n)} \left(\frac{x}{2}\right)^{2n+v} \quad (6)$$

Also for the second exponent $r_2 = -v$, we have

$$J_{-v}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)\Gamma(1-v+n)} \left(\frac{x}{2}\right)^{2n-v} \quad (7)$$

Only put the value of ½ in J_v (x) and - ½ in J_{-v} (x) at the places of v.

Q7: Define the derivative of

$$A(t) = \begin{bmatrix} e^{2t} \\ t^2 \\ 8 \end{bmatrix}$$

Answer: Repeated

Q8: Find the egien values of

$$A \begin{bmatrix} 1 & -1 \\ \frac{4}{9} & \frac{-1}{3} \end{bmatrix}$$

Solution:

Consider the question below.

$A \quad \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$

for eigen values, $|A - \lambda I| = 0$

$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$

$(3-\lambda)(-3-\lambda)+36=0$

$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$

$-9-3\lambda+3\lambda+\lambda^2+36=0$

$-9+\lambda^2+36=0$

$\lambda^2+27=0$

$\lambda=\sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

Q9: bht lamba tha mery sy note ni hoa time thora tha is lia ☹

Q10: Find the auxiliary solution of $x^t = 3x - y - 1$ and $y^t = y + x - 4e^t$

Consult page no 141

Q11: Write down the system of differential equations (5marks)

$\frac{dx}{dt} = 6x + y + 6t$, $\frac{dy}{dt} = 4x + 3y - 10t + 4$

In form of $X' = AX + F(t)$

Solution:

$X' = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} X + \begin{pmatrix} 6t \\ -10t+4 \end{pmatrix}$

==================== PAST PAPERS=========================

Q: An electronic component of an electronic circuit that has the ability to store charge and opposes any change of voltage in the circuit is called

Inductor
Resistor
Capacitor
None of them

Q: If A_o is initial value and T denotes the half-life of the radioactive substance than

$T \quad \frac{1}{2A}$

$\frac{dA}{dt} \quad KA$

$A(T) \quad \frac{A_0}{2}$

None of the above

Q: integrating factor of the given equation $x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)$ is

Xsecx

Cosx

Cotx

Xsinx

Q: Operator method is the method of the solution of a system of linear homogeneous or linear non-homogeneous differential equations which is based on the process of systematic elimination of the
Dependent variables

Independent variable
Choice variable
None of them

Q: If E (t) =0, R =0 Electric vibration of the circuit is called_________

Free damped oscillation
Un- damped oscillation
Over damped oscillation
None of the given

Q: Eigen value of a matrix $\begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$

5, 5

10, 5

25, 5

None

Q: Eigen value of a matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

2,0

1,1

1,2

None

Q: For Eigen values $\lambda \quad 5,5$ of a matrix $A \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$,there exists......... Eigen vectors.

infinite
one
two
three

Q: If a matrix has 1 row and 3columns then the given matrix is called________

Column matrix

Row matrix

Rectangular matrix

None

Q: The general solution of differential equation $\frac{dy}{dx} \quad \frac{x+y}{x}$ .is given by

$e^{\frac{y}{x}} \quad cx$

$e^{\frac{y}{x}} \quad cy$

$e^{\frac{x}{y}} \quad cx$

$e^{-\frac{x}{y}} \quad cx$

Q: The integrating factor of the D.E $\frac{dy}{dx} + y \ln y = ye^{x}$ is

e^{x}

e^{y}

$e^{\frac{1}{x}}$

$e^{\frac{x}{y}}$

Q: For the equation of free damped motion $\frac{dx^2}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = 0$ the roots are $m_1 = -\lambda + \sqrt{\lambda^2 + \omega^2}$ & $m_1 = -\lambda - \sqrt{\lambda^2 + \omega^2}$ if $\lambda^2 - \omega^2 > 0$ Then the equations said to be:

Under damped
Over damped
Critically damped
None of them

Q: For the system of differential equations $\frac{dy}{dt}=2x, \frac{dx}{dt}=3y$ the independent variable is

(Are)

X,t

Y,t

X,y

t

Q: For the system of differential equations $\frac{dy}{dt}=2x, \frac{dx}{dt}=3y$ the dependent variable is

(Are)

X,t

Y,t

X,y

t

Q: $\begin{pmatrix} 4-\lambda & 1 & 0 \\ 0 & 4-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0$ gives

λ 4 of multiplicity of 1

λ 4 of multiplicity of 2

λ 4 of multiplicity of 3

None of the given.

Q: wronksin of x, x^2 is

x^2

x

0

None of the above

a) Matrix A nd value of lembda was given to find the eigen vector? 3 marks.

Answer: (This question is solved by Shining Star as original question was missing so I put it here for reference.)

A= $\begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}$, corresponding Eigen value $\lambda = -2$.

$$\left(\begin{array}{cc|c} -3-(-2) & 1 & 0 \\ 2 & -4-(-2) & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 2 & -2 & 0 \end{array}\right)$$

Add two times row 1 in row 2

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$-k_1 + k_2 = 0$

$k_1 \quad k_2$

Choosing $k_2 = 1$, we get $k_1 = 1$

therefore, eigen vector is $V \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) X'=AX was given to find the eigenvalue and Eigen vector? 5 marks. (This question is solved by Shining Star as original question was missing so I put it here for reference.)

For eigen values consut this question and for eigen vector look at the above.

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

Solution:

$$X' \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix} X$$

$$A \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$$

for eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -9 \\ 4 & -3-\lambda \end{vmatrix} \quad 0$$

$$(3-\lambda)(-3-\lambda)+36=0$$

$$3(-3-\lambda)-\lambda(-3-\lambda)+36=0$$

$$-9-3\lambda+3\lambda+\lambda^2+36=0$$

$$-9+\lambda^2+36=0$$

$$\lambda^2+27=0$$

$\lambda = \sqrt{27}i$ and $-\sqrt{27}i$ are the two complex eigen values

c) Solve DE dy-7dx=0 for initial value f(0)=1? 5 marks.
Answer:

$dy - 7dx = 0$

$dy \quad 7dx$

$\int dy \quad \int 7dx$

$y = 7(x) + c$

$f(0) \quad 1$

$f(0) = 7(0) + c$

$f(0) = 0 + c$

$1 \quad C$

$y = 7x + 1$

d) Find the general solution of 4x^2 y '' + 4xy' (4x^2-25)y=0 (it is the Bessel's Equation a nd same question is given in exercise pg 314 of our handouts)? 5 marks

Answer:

Bessel's differential equation is

$$x^2 y'' + xy' + \left(x^2 - v^2\right) y = 0$$

Example 1

Find the general solution of the equation

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \text{ on } (0, \infty)$$

Solution

The Bessel differential equation is

$$x^2 y'' + xy' + \left(x^2 - v^2\right) y = 0 \qquad (1)$$

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \qquad (2)$$

Comparing (1) and (2), we get $v^2 = \frac{1}{4}$, therefore $v = \pm\frac{1}{2}$

So general solution of (1) is $y = C_1 J_{1/2}(x) + C_2 J_{-1/2}(x)$

Answer:

e) When a function is said to be analytic at any point? 2 marks
Answer:
A function is said to be analytic at point if the function can be represented by power series in (x−a) with a positive radius of convergence.

f) What is the ratio test? (its on pg 264 of our handouts) 5 marks

To determine for which values of x a power series is convergent, one can often use the Ratio Test. The Ratio test states that if

$$\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} c_n (x-a)^n$$

is a power series and

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| |x-a| = L$$

Then:

- The power series converges absolutely for those values of x for which $L < 1$.
- The power series diverges for those values of x for which $L > 1$ or $L = \infty$.
- The test is inconclusive for those values of x for which $L = 1$.

g) What is the formula for radius of convergence? (Its on pg 265 of our hndouts)2 marks

Answer:

$$R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$$

h) Write system of linear differential equations for two variables x and y? (its on pg 333 of our handouts).2 marks
i) write any 3 D.Es of order 2? 3 marks Page no 207
Answer:

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f\left(x\right)$$

j) D.E was given to convert in normal form? 3 marks

Answer:

Reduce the third-order equation

$$2y''' = -y - 4y' + 6y'' + \sin t$$

or

$$2y''' - 6y'' + 4y' + y = \sin t$$

to the normal form.

Solution: Write the differential equation as

$$y''' = -\frac{1}{2}y - 2y' + 3y'' + \frac{1}{2}\sin t$$

Now introduce the variables

$$y = x_1,\ y' = x_2,\ y'' = x_3.$$

Then

$$x_1' = y' = x_2$$
$$x_2' = y'' = x_3$$
$$x_3' = y'''$$

Hence, we can write the given differential equation in the linear normal form

$$x_1' = x_2$$
$$x_2' = x_3$$
$$x_3' = -\frac{1}{2}x_1 - 2x_2 + 3x_3 + \frac{1}{2}\sin t$$

k) Any example of boundary value problem? 2 marks

Consider the function

$$y = 3x^2 - 6x + 3$$

We can prove that this function is a solution of the boundary-value problem

$$x^2 y'' - 2xy' + 2y = 6,$$

$$y(1) = 0, \qquad y(2) = 3$$

Since $\quad \frac{dy}{dx} = 6x - 6, \ \frac{d^2y}{dx^2} = 6$

Therefore $\quad x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6x^2 - 12x^2 + 12x + 6x^2 - 12x + 6 = 6$

Also $\quad y(1) = 3 - 6 + 3 = 0, \quad y(2) = 12 - 12 + 3 = 3$

Therefore, the function ' y satisfies both the differential equation and the boundary conditions. Hence y is a solution of the boundary value problem.

Note: Power series sy ziada NHI tha. Lecture 35 to 45 pr ziada emphsis tha

Q No.2 ------------------5 marks:

Write annihilator operator for x+3xe^ (6x) e ki power 6 xs

$$g(x) = 4e^{2x} - 6xe^{2x}$$

$$(D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = (D^2 - 4D + 4)(4e^{2x}) - (D^2 - 4D + 4)(6xe^{2x})$$

or $\quad (D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = 32e^{2x} - 32e^{2x} + 48xe^{2x} - 48xe^{2x} + 24e^{2x} - 24e^{2x}$

or $\quad (D-2)^2\left(4e^{2x} - 6xe^{2x}\right) = 0$

Therefore, the annihilator operator of the function g is given by

$$L = (D-2)^2$$

We notice that in this case $\alpha = 2 = n$.

Q No.3 ------------------3 marks:

Write the solution of simple harmonic motion in alternative simpler form x(t)=c1coswt+c2sinwt from lec 22 page 199

Answer:

Q No.4 ------------------2 marks:

Define general linear DE of nth order

Answer:

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}}+\cdots+a_1(x)\frac{dy}{dx}+a_0(x)y=g(x)$$

Define elementary row operation.

Answer:

Addition or multiplication of two rows.

The elementary row operations consist of the following three operations

- Multiply a row by a non-zero constant.
- Interchange any row with another row.
- Add a non-zero constant multiple of one row to another row.

Eigenvalue of multiplicity m 3

Answer:

Suppose that m is a positive integer and $(\lambda - \lambda_1)^m$ is a factor of the characteristic equation

$$\det(A - \lambda I) = 0$$

Further, suppose that $(\lambda - \lambda_1)^{m+1}$ is not a factor of the characteristic equation. Then the number λ_1 is said to be an eigenvalue of the coefficient matrix of multiplicity m.

Fundamental of matrix 3

Answer:

Suppose that the a fundamental set of n solution vectors of a homogeneous system $X' = AX$, on an interval I, consists of the vectors

$$X_1 = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}, X_2 = \begin{pmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{n2} \end{pmatrix}, \ldots, X_n = \begin{pmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_{nn} \end{pmatrix}$$

Then a fundamental matrix of the system on the interval I is given by

$$\phi(t) = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{pmatrix}$$

What is determinnant? How to find it.

Determinant of a Matrix

Associated with every square matrix A of constants, there is a number called the determinant of the matrix, which is denoted by $\det(A)$ or $|A|$

Write equation in matrix form.

Find general solution........ 5marks..

Forbenius Theorem.........

$$y = (x - x_0)^r \sum_{n=0}^{\infty} c_n (x - x_0)^n = \sum_{n=0}^{\infty} c_n (x - x_0)^{n+r}$$

5marks

Super position method for vectors
Answer:

$$y = C_1 y_1(x) + C_2 y_2(x)$$

Explain convergence and infinty condition of a infintye sereies.

- If we choose a specified value of the variable x then the power series becomes an infinite series of constants. If, for the given x, the sum of terms of the power series equals a finite real number, then the series is said to be convergent at x.

What does these symbols mean?

Symbol	Meaning
R_{ij}	Interchange the rows i and j.
cR_i	Multiply the ith row by a nonzero constant c.
$cR_i + R_j$	Multiply the ith row by c and then add to the jth row.

Q2. **Solve the system of differential equations** $\frac{dy}{dt} = x, \frac{dx}{dt} = y$ **by systematic elimination.**

Solution:

$\frac{dy}{dt} = x \Rightarrow Dy - x = 0$ ....(i)

$\frac{dx}{dt} = y \Rightarrow -y + Dx = 0$ ....(ii)

Operate (ii) by D, we get

$-Dy + D^2x = 0$ ....(iii)

Add (i) and (iii), we get

$Dy - x = 0$

$\underline{-Dy + D^2x = 0}$

$D^2x - x = 0$

$(D^2 - 1)x = 0$

Auxiliary equation is $m^2 - 1 = 0$

$m = \pm 1$

$\boxed{x(t) \quad c_1e^t + c_2e^{-t}}$

Put this in (i), we get

$Dy - \left[c_1e^t + c_2e^{-t}\right] = 0$

$Dy = c_1e^t + c_2e^{-t}$

Integrate both sides, we get

$\boxed{y(t) = c_1e^t - c_2e^{-t}}$

Q3. **Find a series solution for the differential equation** $y'' + y = 0$ **about** $x_0 = 0$ **such that**

$a_{n+2} = -\frac{a_n}{(n+2)(n+1)}$ $n = 0, 1, 2, \ldots$ **and** $y(x) \quad \sum_{n \; 0}^{\infty} a_n x^n$

Solution:

$$a_{n+2} = -\frac{a_n}{(n+2)(n+1)}\ ; \qquad n = 0,1,2,\ldots$$

For $n=0,\ a_2 = -\frac{a_0}{(0+2)(0+1)} = -\frac{a_0}{2}$

For $n=1,\ a_3 = -\frac{a_1}{(1+2)(1+1)} = -\frac{a_1}{6}$

For $n=2,\ a_4 = -\frac{a_2}{(2+2)(2+1)} = -\frac{a_2}{12} = -\frac{1}{12}\left(-\frac{a_0}{2}\right) = \frac{a_0}{24}$

For $n=3,\ a_5 = -\frac{a_3}{(3+2)(3+1)} = -\frac{a_3}{20} = -\frac{1}{20}\left(-\frac{a_1}{6}\right) = \frac{a_1}{120}$

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \ldots$$

$$y(x) = a_0 + a_1 x + \left(-\frac{a_0}{2}\right)x^2 + \left(-\frac{a_1}{6}\right)x^3 + \left(\frac{a_0}{24}\right)x^4 + \left(\frac{a_1}{120}\right)x^5 + \ldots$$

$$y(x) = a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \ldots\right) + a_1\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots\right)$$

Q4. **Write solution** $X = \frac{4}{3}Cos3t - \frac{5}{3}Sin3t$ **in the form** $X = A.Sin(wt+\phi)$**.**

$$A = \sqrt{\left(\frac{4}{3}\right)^2 + \left(-\frac{5}{3}\right)^2} = \frac{\sqrt{41}}{3}$$

$$\phi = \tan^{-1}\left(\frac{4/3}{-5/3}\right) = 0.6747 \text{ radians}$$

$$x(t) = \frac{\sqrt{41}}{3}\sin(3t + 0.6747)$$

Q5. If the equation $M(x,y)dx + N(x,y)dy = 0$ is not exact,

Case 1:

When $\exists$ an integrating factor $u(y)$, a function of y only. This happens if the expression

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}$$

is a function of y

Case2:

If the given equation is homogeneous and

$$xM + yN \neq 0$$

Then find the integrating factor in both cases.

Solution:

$$u \quad \frac{1}{xM + yN}$$

Q8. **Under which conditions linear independence of the solutions for the differential equation** $y'' + P(x)y' + Q(x)y = 0$ ....(1) **is guaranteed?**

Solution:

Linear independence is guaranteed in case when the Wronskian of the two solutions is not equal to zero.

Q10. **When Frobenius' Theorem is used in Differential Equation** $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ **?**

Solution:

When we have a regular singular point x= x0, then we can find at least one series solution of the form $y = \sum_{n\ 0}^{\infty} c_n \left(x - x_0\right)^{n+r}$, where r is the constant that we will determine after solving the differential equation.

Q12. Define Legendre's polynomial of degree n

Solution:

Legendre polynomial is an n[th] degree polynomial and it is given by the formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left(x^2 - 1\right)^n$$

Q13. **What is the ordinary differential equation and give an example?**

Solution:

A differential equation which only includes ordinary derivatives is known as ordinary differential equation. Some examples of ordinary differential equations include:

$$\frac{dy}{dx} = x^2 + y$$

$$\frac{dy}{dx} = \left(x^2 + 1\right)\left(y^2 + 1\right)$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 0$$